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Analysis of Cyclone Collection Efficiency

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Abstract. *Particle motion in the cyclone outer vortex was analyzed in this paper to establish the force balance differential equation. The Barth's "static particle" theory combined with the force balance equation was applied in the theoretical analyses for the models of cyclone cut-point and collection probability distribution in the cyclone outer vortex. Cyclone cut-points for different dusts were traced from measured cyclone overall collection efficiencies and the theoretical model for calculating cyclone overall efficiency. The traced cut-points indicate that Barth's* d_{50} *model needs to be corrected for particle size distribution (PSD). The cut-point correction factors (K) for 1D3D and 2D2D cyclones were developed through regression fit from traced cut-points The regression results also indicate that cut-points are more sensitive to mass median diameter (MMD) of PSD than to geometric standard deviation (GSD) of PSD. The theoretical overall efficiency model developed in this research can be used for cyclone total efficiency calculation with the corrected* d_{50} *and PSD.*

Keywords. Cyclone collection efficiency, cut-point, particle size distribution, mass median diameter, geometric standard deviation, cut-point correction factor

Introduction

Cyclones, as the most cost-effective air pollution abatement device for particulate matter (PM) removal, have been studied for decades. To design or regulate a cyclone system, it is essential to accurately predict cyclone performance. Unfortunately, previous efforts to develop a theoretical model for cyclone fractional efficiency have not been completely accurate. In the past, cyclones have been designed by the classical cyclone design (CCD) process (Cooper and Alley, 1994), which was developed by Lapple (1951). However there are several problems associated with this design procedure. First of all, the CCD process does not consider the cyclone inlet velocity in developing cyclone dimensions. It was reported (Parnell, 1996) that there is an "ideal" inlet velocity for the different cyclone designs for optimum cyclone performance. Secondly, the CCD does not predict correct number of turns for different type cyclones The overall efficiency predicted by the CCD process is incorrect because of the inaccurate fractional efficiency curve generated by the CCD process (Kaspar et al. 1993).

Cyclone fractional efficiency curves relate percent efficiency to the particle diameter. It is generally assumed that cyclone fractional efficiency curves (FEC) are accurately represented by a lognormal relationship defined by the cut-point (d_{50}) and slope of the curve ($d_{84.1}/d_{50}$). The cut-point is the aerodynamic equivalent diameter of the particle collected with 50% efficiency and the slope of the fractional efficiency curve is the ratio of the 84.1% and 50% particle sizes ($d_{84.1}/d_{50}$) or the ratio of the 50% and 15.9% particle sizes ($d_{50}/d_{15.9}$) from the fractional efficiency curve. It is also assumed in the CCD process that once defined, the FEC is not significantly affected by the particle size distribution (PSD) of particulate matter (PM) entering. However, previous research at Texas A&M University (TAMU) (Wang et al. 2000) indicated that the cyclone FEC's are not independent of inlet PSD's. As a result of this research, the shifting of cut-point with the change of inlet PSD has been reported.

Study of the particle collection mechanism in the outer vortex is a way to understand the relationship between the cyclone performance characteristics and the design and operating parameters, and then to develop a mathematical model to predict the cyclone performance. This study is based upon the knowledge of the flow pattern in the cyclone outer vortex. Many investigations have been made to determine the flow pattern (velocity profile) in a cyclone irrotational fluid field. Shepherd and Lapple (1939) reported that the primary flow pattern consisted of an outer spiral moving downward from the cyclone inlet and an inner spiral of smaller radius moving upward into exit pipe. These flows are known as outer vortex and inner vortex. Transfer of fluid from the outer vortex to the inner vortex apparently begins below the bottom of the exit tube and continues down into the cone to a point near the dust outlet at the bottom of the cyclone. Shepherd and Lapple concluded from streamer and pitot tube determinations that the radius marking the outer limit of the inner vortex and the inner limit of the outer vortex was roughly equal to the exit duct radius (Figure 1). Ter Linden (1949) measured the details of the flow field in a 14 in cyclone. He reported that the interface of the inner vortex and the outer vortex occurred as a radius somewhat less than that of the exit duct in the cylindrical section of the cyclone and approached the centerline in the conical section.

The length of the inner vortex core also referred to as the cyclone effective length (Z_o in Figure 1) is determined by the diameter (D_o in Figure 1) of the interface of the inner vortex and the outer vortex. Cyclone effective length does not necessarily reach the bottom of the cyclone (Leith and Mehta, 1973). When the cyclone effective length is shorter than the cyclone physical length, the space between the bottom of the vortex and the bottom of the cyclone will not be used for particle collection. On the other hand, if the effective length is longer than the cyclone physical length, the vortex will extend beyond the bottom of the cyclone, and a dust re-entrance problem will occur.

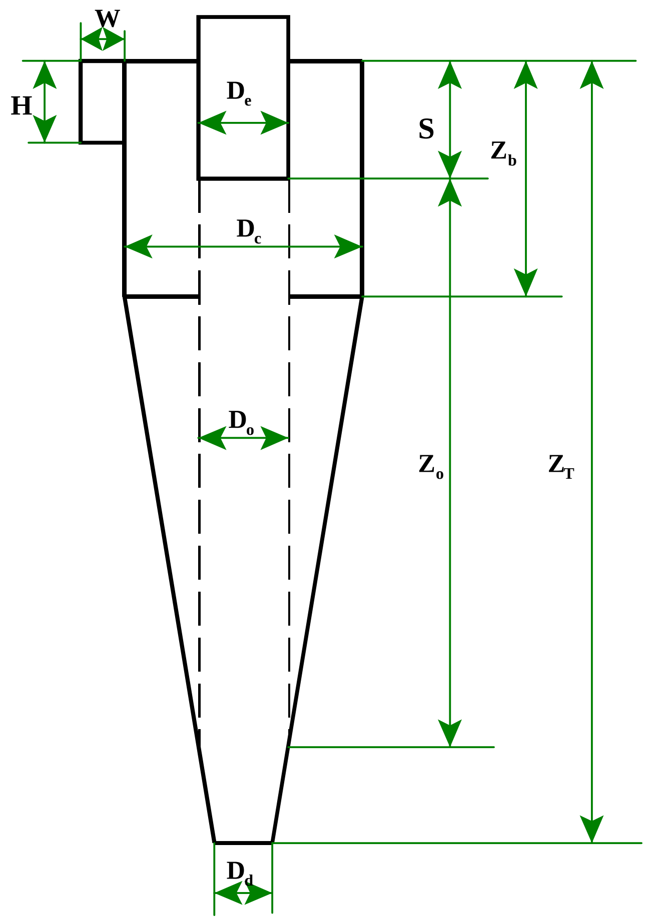

Figure 1. Cyclone inner vortex core dimensions

The velocity profile in a cyclone can be characterized by three components (tangential, axial and radial). The tangential velocity is the dominant component. It also determines the centrifugal force applied to the air stream. Research results of Shepherd and Lapple (1939), Ter Linden (1949) and Leith and Licht (1972) indicated that air stream tangential velocity in the annular section (at the same cross-sectional area) of the cyclone could be determined by equation 1.

$$V_t * r^n = C_1 \quad \text{……………………………………………(1)}$$

where:

V_t = tangential velocity,
r = air stream radius,
n = 0.5 (in outer spiral) (Shepherd and Lapple, 1939), and
C_1 = a numerical constant.

For simplicity, it is assumed that V_t equals the average air stream inlet velocity V_{in} when r equals the radius of the cyclone wall (R), that is:

$$V_t * r^{0.5} = V_{in} * R^{0.5}$$

$$V_t = \left(\frac{R}{r}\right)^{0.5} * V_{in} \quad \text{…..………………………………}(2)$$

where:

V_t = tangential velocity,
V_{in} = inlet velocity,
R = the radius of the cyclone wall, and
r = air stream radius.

Theoretical Analysis of Cyclone Collection Efficiency

The particle motion in the cyclone outer vortex can be modeled by Newton's law as follows:

$$m_p \frac{d\vec{V}}{dt} = \sum \vec{F} \quad \text{………………………..………… (3)}$$

where:

V = particle velocity,
F = external force, and
m_p = particle mass.

For flows in which Stokes law applies, the drag force on a spherical particle may be determined by the Stokes law (equation 4) and the centrifugal force is determined by the equation 5.

$$F_D = 3\pi\mu d * (V_r - V_{gr}) = 3\pi\mu d * (\frac{dr}{dt} - V_{gr}) \quad \text{……………..……. (4)}$$

where:

F_D = drag force acting on the particle,
μ = air viscosity,
V_r = particle radial velocity, and
V_{gr} = air stream radial velocity.

$$F_c = m\vec{a} = m\vec{a}_r = \frac{\pi * d^3}{6} * \rho_p * \left(\frac{d^2r}{dt^2} - r(\frac{d\theta}{dt})^2\right) \text{……………….….(5)}$$

where:

F_c = centrifugal force acting on the particle in radial direction,
m = particle mass,
ρ_p = particle density, and
d = particle diameter.

In the cyclone outer vortex fluid field, there are only two forces (centrifugal force F_c & drag force F_D) acting on the particle in the radial direction. When $F_c > F_D$, the particle moves towards the cyclone wall to be collected. Whereas, when $F_c < F_D$, the particle will move to the inner vortex and then to penetrate the cyclone. The force balance ($F_c = F_D$) gives the particle a 50% chance to penetrate and a 50% chance to be collected. The force balance differential equation can be setup by letting equation 4 equal to equation 5, i.e. $F_c = - F_D$, it yields equation 6.

$$\left(\frac{d^2r}{dt^2} - r(\frac{d\theta}{dt})^2\right) = -\frac{18\mu}{\rho_p * d^2} * \left(\frac{dr}{dt} - V_{gr}\right) \text{………..…………..…(6)}$$

where:

$\frac{dr}{dt}$ = particle radial velocity (V_r),

$r\frac{d\theta}{dt}$ = particle tangential velocity (the same as air tangential velocity V_t can be determined by the equation 2),

V_{gr} = air stream radial velocity,

μ = air viscosity,

ρ_p = particle density, and

d = particle diameter.

The solution of the particle force balance differential equation above gives the particle radial trajectory. This trajectory is the critical path in the radial direction and is a function of particle diameter. As mentioned above, the force balance gives 50% collection probability. In other words, the particle diameter is d_{50} when the forces on a particle are in equilibrium on the critical path. The force balance differential equation yields a d_{50} distribution in the cyclone outer vortex (Wang et al. 2002).

Barth (1956) Model For Cyclone D_{50} – One Solution To Equation 6

Stairmand (1951) and Barth (1956) first developed the "static particle" theory for the analysis of cyclone collection efficiency in the 50's. Since then, this static particle theory based upon the force balance analysis has been adopted by many other researchers in their theoretical analyses for characterizing the cyclone performance. Basically the static particle theory suggested that the critically sized particle (d_{50}) is smaller than the smallest particle, which is collected, and larger than the largest particle that penetrates the cyclone. Particles with diameter of d_{50} theoretically suspend in the outer vortex due to the force balance. Even though there is a d_{50} distribution in the outer space, only those d_{50} on the interface of inner vortex and outer vortex will characterize the cyclone performance and represent the cut-point of the cyclone. To solve the force balance differential equation 6, Barth made several assumptions. First, the particle radial velocity was assumed to be zero because of static status. It was also assumed that air uniformly leaked from the outer vortex to the inner vortex. So, the air inwards radial velocity was determined by the equation 7.

$$V_{gr} = \frac{Q}{\pi * D_o * Z_o} \quad \text{………………..………………} \quad (7)$$

where:

V_{gr} = air stream radial velocity,

Q = volume flow rate through the cyclone,

D_o = inner vortex core diameter, also known as interface diameter, and

Z_o = vortex length, also known as cyclone effective length.

Substituting the equation 7 into equation 6, Barth developed a mathematical model for the cyclone cut-point as follows:

$$d_{50} = \sqrt{\frac{9 * \mu * Q}{\pi * \rho_p * Z_o * V_{in}^2}} \quad \text{…………….……………} \quad (8)$$

where:

V_{in} = cyclone inlet velocity,

μ = air viscosity,

ρ_p = particle density,

Q = volume flow rate through the cyclone,

D_o = inner vortex core diameter, also known as interface diameter, and

Z_o = vortex length, also known as cyclone effective length.

There is an inherent problem associated with the force balance analyses. The mathematical model was only based upon the analysis for the individual particle. It did not consider the particle size distribution of the inlet PM. However, the cyclone cut-point changes with the PSD of inlet PM. So, a correction factor, which is function of PSD, is needed.

The Particle Collection Probability Distribution In The Outer Vortex

Based on the analyses above, the d_{50} distribution gives the critical separation diameter (d_{50}) at any point P(r, z) in the outer vortex. At the point P(r, z), if the particle diameter d > d_{50}, the particle will move to the wall and be collected, whereas if the particle diameter d < d_{50}, the particle will move to the inner vortex and penetrate. For a given inlet particle size distribution, the ratio of all the particles larger than d_{50} to the total inlet particles is the particle collection probability at the point P(r, z). If it is assumed that the inlet particle size distribution is a log-normal distribution with mass median diameter (MMD) and geometric standard deviation (GSD), then equation 12 can be used to determine the particle collection probability at any point P(r, z) in the outer vortex.

$$P(d) = \int_{d_{50}}^{\infty} \frac{1}{\sqrt{2\pi} d \ln(GSD)} \exp[-\frac{(\ln(d) - \ln(MMD))^2}{2(\ln(GSD))^2}] dd \quad \text{………………..(9)}$$

where:

P (d) = particle collection probability at the any point P(r, z) in the outer vortex,
d_{50} = the critical separation diameter with a 50% collection probability at the point P(r, z),
d = particle diameter,
MMD = mass median diameter of the inlet particle size distribution, and
GSD = geometric standard deviation of the inlet particle size distribution.

When the critical separation diameter (d_{50}) on the interface is used in the equation 9, the integration yields the cyclone total efficiency. In other words, equation 9 is the theoretical model for calculating cyclone overall efficiency.

Tracing Cyclone Cut-Points (D_{50}) From Measured Cyclone Overall Efficiency And PSD

To determine the relationship of cyclone cut-points and the PSD's, equation 9 was used to theoretically trace the d_{50} from measured cyclone total efficiency with five kinds of dust (Wang et al. 2000). The traced d_{50} for 1D3D and 2D2D cyclones are listed in the table 1.

Table 1. Traced cut-points (d_{50}) from measured overall efficiency and PSD for 1D3D and 2D2D cyclones

		PSD	1D3D		2D2D	
	ρ_p	MMD / GSD	Measured η_{total}	Traced $\mathbf{d_{50}}$	Measured η_{total}	Traced $\mathbf{d_{50}}$
Dust A	1.77	20 / 2.0	99.7%	3.00	99.6%	3.20
Dust B	1.82	21 / 1.9	99.3%	4.30	98.9%	4.82
Dust C	1.87	23 / 1.8	99.7%	4.50	99.6%	4.80
Cornstarch	1.52	19 / 1.4	99.3%	8.25	99.2%	8.50
Fly ash	2.73	13 / 1.7	96.8%	4.85	95.5%	5.25

- PSD: particle size distribution
- Dusts A, B, and C are fine cotton gin dusts from different ginning processing streams. The dusts had been passed through a screen with 100 mm openings.
- MMD: mass median diameter (μm) of PSD
- GSD: geometric standard deviation

- ρ_p : particle density (g/cm^3)
- Measured η_{total}: measured overall cyclone efficiency from previous research (Wang et al. 200).
- Traced d_{50}: d_{50} (μm) obtained from equation 9 by setting P (d) equal to the overall efficiency.

It is observed from table 1 that the cut-point of a cyclone changes with the PSD. This is the same observation reported by Wang et al. (2000) from the previous experimental research. Table 2 shows the comparison of the traced d_{50}'s and the experimental d_{50}. The results listed in the table 2 suggest that cyclone cut-point is function of MMD and GSD of inlet dust PSD. When the GSD larger than 1.5, the cut-points decrease with the increase of MMD (see gin dust vs. fly ash), whereas the cut-points increases with the increase with MMD when the dust GSD is less than 1.5.

Table 2. Comparison of the traced cut-points vs. experimental cut-points

	1D3D		2D2D	
	Traced $\mathbf{d_{50}}$	Experimental $\mathbf{d_{50}}$	Traced $\mathbf{d_{50}}$	Experimental $\mathbf{d_{50}}$
Dust A	3.00	2.50	3.20	2.74
Dust B	4.30	3.55	4.82	3.75
Dust C	4.50	3.34	4.80	3.60
Cornstarch	8.25	---	8.50	---
Fly ash	4.85	4.25	5.25	4.40

- Traced d_{50}: d_{50} (μm) obtained from equation 9 by setting P (d) equal to the overall efficiency
- Dusts A, B, and C are fine cotton gin dusts from different ginning processing stream. The dusts had been passed through a screen with 100 mm openings
- Experimental d_{50} (μm) were determined from experimental fractional efficiency curves calculated from experimental measurements of inlet and outlet concentration and PSD's (Wang et al. 2002)
- No experimental d_{50} available for cornstarch.

Corrected Model For Cyclone Cut-Point And Determination Of Cyclone Overall Efficiency

The comparisons of cut-points obtained by using Barth model (equation 8) and the traced cut-points by using equation 9 and measured overall efficiencies for the different dust are shown in the table 3. The cut-points from Barth model do not change with PSD, which is not consistent with the experimental research.

Table 3. Comparison of the traced cut-points vs. cut-points obtained from a theoretical model

	1D3D		2D2D	
	Traced $\mathbf{d_{50}}$	Barth model $\mathbf{d_{50}}$ (AED)	Traced $\mathbf{d_{50}}$	Barth model $\mathbf{d_{50}}$ (AED)
Dust A	3.00	3.58	3.20	3.46
Dust B	4.30	3.58	4.82	3.46
Dust C	4.50	3.58	4.80	3.46
Cornstarch	8.25	3.58	8.50	3.46
Fly ash	4.85	3.58	5.25	3.46

- Traced d_{50}: d_{50} (μm) obtained from equation 9 by setting P (d) equal to the overall efficiency
- Dusts A, B, and C are fine cotton gin dusts from different ginning processing stream. The dusts had been passed through a screen with 100 mm openings
- Barth model d_{50}'s are determined by equation 8

It is necessary to introduce a cut-point correction factor (K) to modify the theoretical d50 model to quantify the effect of PSD on the cut-point calculation. Table 4 lists K values based on Barth's d_{50}'s and traced d_{50}'s. It is obvious that the K value is a function of MMD and GSD. A regression analysis was performed to determine the relationship of K and MMD and GSD. Equations 10 and 11 show the results of regression fit based upon the data in the table 4. It is noticed from the regression fit that the GSD has greater effect on K than MMD. In other words, the cut-points is more sensitive to GSD than to MMD.

Table 4. Cut-point correction factor for 1D3D and 2D2D cyclones with different dusts

	PSD		Cut-point correction factor **(K)**	
	MMD	GSD	1D3D	2D2D
Dust A	20	2.0	0.84	0.92
Dust B	21	1.9	1.20	1.39
Dust C	23	1.8	1.26	1.39
Cornstarch	19	1.4	2.31	2.46
Fly ash	13	1.7	1.36	1.52

- PSD: particle size distribution
- Dusts A, B, and C are fine cotton gin dusts from different ginning processing stream. The dusts had been passed through a screen with 100 mm openings.
- MMD: mass median diameter (μm) of PSD
- GSD: geometric standard deviation.

$$K_{1D3D} = 5.3 + 0.02 * MMD - 2.4 * GSD \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

where:

K_{1D3D} = cut-point factor for 1D3D cyclone (dimensionless),
MMD = mass median diameter (μm) of PSD, and
GSD = geometric standard deviation of PSD.

$$K_{2D2D} = 5.5 + 0.02 * MMD - 2.5 * GSD \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

where:

K_{2D2D} = cut-point factor for 1D3D cyclone (dimensionless),
MMD = mass median diameter (μm) of PSD, and
GSD = geometric standard deviation of PSD.

Putting the cut-point correction factor into Barth d_{50} model, the cyclone cut-point can be determined by the equation 12 which is referred to as the corrected cut-point model.

$$d_{50} = K * \sqrt{\frac{9 * \mu * Q}{\pi * \rho * Z_o * V_{in}^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.... (12)$$

where:

d_{50} = cyclone cut-point,
K = cut-point correction factor (K= K_{1D3D} for 1D3D cyclone design and K = K_{2D2D} for 2D2D cyclone design),
V_{in} = cyclone inlet velocity,
μ = air viscosity,
ρ = particle density,
Q = volume flow rate through the cyclone,
D_o = inner vortex core diameter, also known as interface diameter, and
Z_o = vortex length, also known as cyclone effective length.

The theory and the methodology used in this research for correcting the cut-point model indicate that it is not necessary to develop a fractional efficiency curve to calculate the cyclone overall efficiency. The process for calculating cyclone efficiency can be summarized as the following steps:

1. Calculate the cut-point correction factor for the different cyclone design and the given PSD (MMD&GSD).
2. Determine the cut-point using the corrected d_{50} model (equation 12).
3. Determine the overall efficiency by integrating equation 9 based up the corrected cut-point and PSD (MMD&GSD).

Conclusion

Particle motion in the cyclone outer vortex was analyzed in this paper to establish the force balance differential equation. The Barth's "static particle" theory combined with the force balance equation was applied in the theoretical analyses for the models of cyclone cut-point and collection probability distribution in the cyclone outer vortex. Cyclone cut-points for different dusts were traced from measured cyclone overall collection efficiencies and the theoretical model for the cyclone overall efficiency calculation. Based upon the theoretical study in this research the following conclusions are obtained:

1. The traced cut-points indicate that cyclone cut-point is the function of dust PSD (MMD&GSD).
2. Theoretical d_{50} model (Barth model) needs to be corrected for PSD.
3. The cut-point correction factors (K) for 1D3D and 2D2D cyclone were developed through regression fits from theoretically traced cut-points and experimental cut-points.
4. The corrected d_{50} is more sensitive to GSD than to MMD.
5. The theoretical overall efficiency model developed in this research can be used for cyclone total efficiency calculation with the corrected d_{50} and PSD. No fractional efficiency curves are need for calculating total efficiency.

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