# UNCERTAINTY ASSOCIATED WITH PARTICULATE MATTER CONCENTRATION MEASUREMENTS FROM AGRICULTURAL OPERATIONS


**Jacqueline E. Price, Ron E. Lacey, Bryan W. Shaw, Calvin B. Parnell, Jr.**
**Center for Agricultural Air Quality Engineering and Science**
**Texas A&M University**
**College Station, TX**



## Abstract

*Gravimetric measurement of particulate matter (PM) concentrations in ambient environments is the basis for regulation of PM fractions (i.e. $PM_{10}$ and $PM_{2.5}$) under the Federal Clean Air Act. While the measurement is straight forward, inherent elements of uncertainty enter the analysis, resulting in much larger uncertainty in the concentration calculation. This paper discusses the importance of uncertainty approximation and analyzes the uncertainty inherent in a gravimetric PM concentration measurement. Utilizing a first order Taylor Series approximation and analytical derivatives, the overall system uncertainty is computed. Additionally, this paper uses a sensitivity analysis of the contributing uncertainty elements in order to identify the most critical measurements and their implications on the calibration, operation, and design of PM samplers.*


## Introduction

Gravimetric measurement of particulate matter (PM) concentration in ambient environments is the basis for regulation of PM fractions (i.e. $PM_{10}$ and $PM_{2.5}$) under the Federal Clean Air Act. In cotton ginning, particulate matter (PM) is considered the primary emitted air pollutant. In general, PM emissions from gins processing picker-harvested cotton are typically less than those of gins processing stripper-harvested cotton, and the PM emissions from the ginning of the first harvest of cotton are generally less than the PM emissions from later harvests (U.S. EPA, 1995). Additionally, data shows that approximately 37% of the total PM emitted from cotton ginning (following PM control systems) is $PM_{10}$, which describes particulate matter with an aerodynamic equivalent diameter less than or equal to 10 μm (U.S. EPA, 1995). However, we know that assuming a lognormal particle size distribution of the PM in the air with a typical cotton gin dust mass median diameter of 20 μm and GSD of 2.0, the mass of $PM_{10}$ on the filter equals approximately 16% of the total suspended particulate matter measured (Wang, 2000).

While these PM measurements are straight forward, numerous elements of uncertainty can enter the analysis, resulting in much larger uncertainty in the concentration calculation. This discussion covers the incorporation of uncertainty analysis in gravimetric measurement of particulate matter.

A measurement of a variable can only provide a deterministic estimate of the quantity being measured; thus, it can only be considered complete when supplemented by a quantitative statement of the inaccuracies surrounding the measurement. Therefore, proper experimental planning and design requires an understanding of the errors inherent in these measurements so that the experimenter can have some degree of certainty in the final measurements and calculations.

*Uncertainty* can be defined as the statistical representation of the reliability associated with a specific set of measurements (Yegnan et al., 2002). Uncertainty can also be described as the possible set of values on a given measurement and can be considered a statistical variable (Kline, 1985). The term *error* takes on a slightly different definition. The total error, $\delta$, is the difference between the measured value and the true value of the quantity being measured. It can also be thought of as the sum of the *systematic error* and the *random error*, $\delta = \beta + \varepsilon$, where $\beta$ is the systematic error and $\varepsilon$ is the random error (ANSI/ASME, 1998). This is illustrated by Figure 1.

Systematic error, $\beta$, also known as fixed error or bias, is defined as the constant element of the total error, $\delta$; therefore, this error value remains constant for each measurement. Random error, $\varepsilon$, also known as repeatability error or precision error, is the random error element of the total error, thus each measurement takes on a different value for this part of the total error measurement (ANSI/ASME, 1998). Thus, the term error refers to a fixed quantity, and it cannot be considered a statistical variable.

Many of the current methods of estimating the uncertainty surrounding experimental results are based upon an analysis by Kline and McClintock (1953). With the goal in mind of determining the effects of each potential measurement error, they proposed a process which considers the impact of these individual uncertainties, commonly referred to as the propagation of uncertainty (Kline and McClintock, 1953). This process involves a Taylor series approximation to estimate the uncertainty in various circumstances.

## Objectives

The objectives of this uncertainty analysis are:

1. To determine the uncertainty surrounding the gravimetric particulate matter (PM) concentration using a first-order Taylor series approximation method.
2. To identify the most critical measurements and their implications on the calibration, operation, and design of PM samplers using a sensitivity analysis.

## Methodology

The impact of the individual uncertainties of each primary measurement in an experiment on the total systematic uncertainty of the experiment must be approximated. This idea is commonly referred to as the law of propagation of uncertainty (ISO, 1995). The uncertainties from the individual independent variables propagate through a data reduction equation into a resulting overall measurement of uncertainty as demonstrated in Figure 2 (Coleman & Steele, 1999).

## Primary Systematic Uncertainty Determination

Manufacturers specify the accuracy of their respective measurement instrument, and this information is used in this analysis as the value for the systematic uncertainty of the measuring device. This accuracy specification takes into account various factors such as linearity, gain, and zero errors (Coleman & Steele, 1999). All of the uncertainty values used in this discussion except for that of the pressure drop across the orifice meter ($\Delta P_a$) were obtained from the specifications on the manufacturers' data sheets. The uncertainty value given by the manufacturer must include any sensor or transducer bias in the system. In the case of the $\Delta P_a$ reading from the Hobo instrument, the bias in both the pressure transducer and the Hobo data logger must be accounted for.

### Uncertainty Propagation Calculation

With the individual systematic uncertainties now determined, the propagated systematic uncertainty can be calculated. Assuming that all individual uncertainties are at the same confidence level (95% confidence interval or 20:1 odds in this instance), let $Y$ be a function of independent variables $x_1, x_2, x_3, \ldots, x_n$. Therefore, the data reduction equation for determining Y from each $x_i$ is

$$Y = Y(x_1, x_2, \ldots, x_n) \qquad [1],$$

Furthermore, let $\omega$ be defined as the systematic uncertainty in the result and $\omega_1$, $\omega_2$, ... , $\omega_n$ as the systematic uncertainties in each of the above independent variables. Given the same confidence interval on each of the independent (uncorrelated) variables, the resulting systematic uncertainty of Y, $\omega_Y$, can be calculated as the positive square root of the estimated variance, $\omega_y^2$, from the following equation (Holman, 2001)

$$\omega_Y = +\sqrt{\omega_Y^{\,2}} \qquad [2],$$

where the variance, $\omega_y^2$, is calculated by

$$\omega_Y{}^2 = \left(\frac{\delta Y}{\delta x_1}\omega_1\right)^2 + \left(\frac{\delta Y}{\delta x_2}\omega_2\right)^2 + \ldots + \left(\frac{\delta Y}{\delta x_n}\omega_n\right)^2 \qquad [3],$$

or

$$\omega_Y{}^2 = (\theta_1\omega_1)^2 + (\theta_2\omega_2)^2 + \ldots + (\theta_n\omega_n)^2 \qquad [4],$$

where θ , the *sensitivity coefficient*, is defined as

$$\theta_i = \frac{\delta Y}{\delta x_i} \qquad [5].$$

**Gravimetric Sampling Governing Equations**

The concentration of particulate matter (PM) in the air can be measure by gravimetric means, where the PM in the air is captured on a filter and then weighed. The particulate matter concentration is a function of the mass of PM collected in a known volume of air as indicated in equation 6 below.

$$C = \frac{W}{V} \qquad [6],$$

where C is the concentration, W is the mass of $PM_{10}$ collected on the filter, and V is the total volume of air through the system during the entire time of sampling. Both W and V are calculated quantities from other measurements. Therefore, these quantities must be reduced to basic measurements as seen in Figure 3.

First, the mass on the filter, W, is necessary. Assuming a lognormal particle size distribution of the PM in the air with a typical cotton gin dust mass median diameter of 20 μm and GSD of 2.0, the mass of $PM_{10}$ on the filter equals approximately 16% of the total suspended particulate matter measured (Wang, 2000). Therefore, the mass of $PM_{10}$ on the filter is calculated by equation 7.

$$W = 0.16 * (W_f - W_i) \qquad [7],$$

where $W_f$ is the weight of the filter and PM after the sampling period and $W_i$ is the weight of the bare filter before the sampling period. These filters are weighed three times before and after sampling under controlled environmental conditions (relative humidity and temperature has an impact on the accuracy), and the mean of each of these three measurements is used. Both $W_f$ and $W_i$ are primary measured quantities, so no further reduction is necessary.

The total volume of air in $ft^3$, V, used during the sampling time is determined by equation 8.

$$V = Q * \Theta \qquad [8],$$

where Q is the volumetric flow rate in cfm and θ is the elapsed time of the test in minutes. The elapsed time of the test, θ, is a measured quantity; however, Q is not. So, Q must be evaluated further. Each gravimetric sampler uses a fan or pump to draw air downward through the filter. The fan/pump setup includes an orifice meter in the line to the sampler in order to calculate the volumetric flow rate of air through the tube. The volumetric flow rate in cfm, Q, is calculated from the pressure drop across an orifice meter as in the following equation, which is derived from Bernoulli's equation (Sorenson and Parnell, 1991).

$$Q = 5.976 * k * (D_0)^2 * \sqrt{\frac{\Delta P_a}{\rho_a}} \quad [9],$$

where k is a calibration constant for the orifice meter, $\Delta P_a$ is the measured pressure drop across the orifice meter in inches of water using a transducer output to a data logger to record the instantaneous pressure drop across the orifice meter, $\rho_a$ is the mean air density in lbs*ft$^{-3}$, and $D_0$ is the diameter of the orifice in inches determined by the end mill specifications. For field sampling measurements, the gas used is air where the air density in lbs*ft$^{-3}$ can be estimated by equation 10 (Cooper and Alley, 1994).

$$\rho_a = \left[\frac{P_a - RH * P_s}{0.37 * (460 + T)}\right] + \left[\frac{RH * P_s}{0.596 * (460 + T)}\right] \quad [10],$$

where $P_s$ is the saturated vapor pressure in lbs*in$^{-2}$ at T (Engineering Toolbox, 2003), T is the dry bulb temperature of the air in degrees Fahrenheit, and RH is the relative humidity fraction of the air. In three of the four examples that follow, the value of k is determined against a laminar flow element (LFE) of greater precision and accuracy than the orifice meter, where the value of k is given by equation 11.

$$k = \frac{Q_{LFE}}{5.976 * (D_0)^2 * \sqrt{\frac{\Delta P_c}{\rho_c}}} \quad [11],$$

where $Q_{LFE}$ is the flow given by the LFE (ft$^3$*min$^{-1}$), $\rho_c$ is the density of the air during calibration (lbs*ft$^{-3}$), and $\Delta P_c$ is the pressure drop across the orifice meter during calibration in inches of water. In the low volume example, the reading from a mass flow meter ($Q_{massflowmeter}$) is used in lieu of $Q_{LFE}$ in equation 11 (to determine the k value). The density of the air during calibration, $\rho_c$, is calculated using the same equation as $\rho_a$, (refer to equation 10).

## Results and Discussion

### Sensitivity Coefficient Determination

In order to evaluate the effect of each primary measurement on the final concentration measurement, the sensitivity must be calculated with respect to each of these primary measurements. The sensitivity coefficient for each element of gravimetric sampling system is based on equation 5. In order to determine the sensitivity coefficients, the systematic uncertainty of each instrument is necessary. Table 1 specifies the instruments used for each measurement as well as the related systematic uncertainty as provided in the manufacturer's specifications. These uncertainty values are assumed to be at a 95% confidence interval (2 standard deviations from the mean, also referred to as 20:1 odds). Literature identifies this as a Type B analysis in which the evaluation of systematic uncertainty is based upon scientific judgment and manufacturers' specifications (NIST, 1994).

With this systematic uncertainty information, the sensitivity coefficient for each variable in equations 6-11 is determined using partial differential equations (as described by equation 5). These partial differentials can be found in Appendix A.

## Sensitivity & Uncertainty Analysis

To determine the most sensitive input parameters with respect to the output particulate matter concentration, a sensitivity analysis must be performed on the uncorrelated primary measurements

(Yegnan et al, 2002). The information obtained from the sensitivity analysis is used to obtain the uncertainty in the particulate matter concentration calculation. Additionally, this information helps the experimenter identify the most influential sources of uncertainty. This proves to be important when the amount of uncertainty in the final computation needs to be reduced by identifying these influential sources of uncertainty.

This analysis evaluates the $PM_{10}$ concentrations in four situations: the high volume sampling technique (Q ~ 50 cfm, which is the midpoint of the U.S. EPA defined appropriate operating flow rates; Q ~ 39 cfm and Q ~ 60 cfm, which are the upper and lower limit flow rates as defined by the U.S. EPA) and low volume sampling technique (Q ~ 0.6 cfm ~ 1 $m^3$/min) used by the Texas A&M Center for Agricultural Air Quality Engineering & Science (CAAQES). It is important to note that the sampling instrumentation used by CAAQES has less uncertainty and variability associated with each piece of instrumentation than the approved EPA sampling instrumentation.

Each portion of Table 2 is a summary of the sensitivity of each independent parameter contributing to the final particulate matter concentration. This information is derived from a model in Microsoft Excel as provided in Appendix B. Using the process defined in the methods section, the sensitivities of each of the parameters are calculated based on equation 5. The uncertainty of each secondary measurement (the propagation of the primary measurements) is determined by the process as described in equations 3 and 4. These secondary uncertainties include not only the uncertainty in the concentration measurement ($\omega_C$) but also the uncertainty in the mass on the filter ($\omega_W$), the volume of air ($\omega_V$), the volumetric flow rate of air ($\omega_Q$), the density of the air during the sampling period ($\omega_{\rho a}$), the density of the air during the orifice meter calibration ($\omega_{\rho c}$) and the k value across the orifice meter ($\omega_k$). Ultimately, the model calculates the amount of impact of each parameter on the total uncertainty in the final concentration calculation. It is important to note that simply adding up the impact of each parameter on the final uncertainty will yield a value much larger than 100%. However, if the parameters representing the primary measurements are summed ($\Delta P_a$, $T_a$, $P_a$, $RH_a$, $P_{sata}$, $Q_{LFE}$, $D_0$, $\Delta P_c$, $T_c$, $P_c$, $RH_c$, $P_{satc}$), then the Percentage of Total Uncertainty results in 100% of the total uncertainty.

The following scenario evaluations are included in Tables 2 and 3 (with the calculations included in Figures 4 – 7):

- TAMU Gravimetric Sampling – Q ~ 0.6 cfm (1 $m^3$/hr)
- TAMU Gravimetric Sampling – Q ~ 39 cfm
- TAMU Gravimetric Sampling – Q ~ 50 cfm
- TAMU Gravimetric Sampling – Q ~ 60 cfm

Table 3 displays the overall concentration uncertainty for each of the scenarios, while Table 2 breaks down the uncertainty into the contribution of each measurement to the total uncertainty.

In all four scenarios, it's important to note that the leading contributor to the uncertainty in the final concentration calculation is the pressure drop across the orifice meter. If we are to seek a higher degree of certainty in our final concentration calculation, then the optimal decision would be to decrease the uncertainty in the pressure drop across the orifice meter measurement.

## Conclusions

A measurement of a variable can only provide a deterministic estimate of the quantity being measured; thus, it can only be considered complete when supplemented by a quantitative statement of the inaccuracies surrounding the measurement. Thus, it is extremely important that all scientific measurements and calculations include a statement of uncertainty. This analysis uses a first order Taylor Series approximation to determine the total uncertainty surrounding the PM concentration for four gravimetric sampling scenarios.

In addition to determining the total uncertainty, the most critical measurements in gravimetric sampling of PM are identified using a sensitivity analysis. In evaluating the uncertainty surrounding each measurement and the impact on the total uncertainty in the final calculation, it is notable that the pressure drop across the orifice meter during the test as well as during calibration accounts for approximately 60% - 80% of the total uncertainty in each of the four examples. With this knowledge, the experimenter has identified the optimal part of the measurement process to focus on to effectively reduce the total uncertainty in the experiment, if desired.

Thus, this analysis has provided a systematic method of determining which instruments in the process need to be improved on in terms of reducing overall uncertainty by using a Taylor Series approximation approach based off of the pioneering research by Kline and McClintock in 1953. An uncertainty analysis should be included in every single experimental procedure!

## References


American National Standards Institute/American Society of Mechanical Engineers (ANSI/ASME). *Test Uncertainty*, Performance Test Code 19.1 – 1998. New York, NY: ASME.

Coleman, Hugh W., and W. Glenn Steele. 1999. *Experimentation and Uncertainty Analysis for Engineers*. 2nd ed. New York, NY: John Wiley & Sons.

Cooper, C. David, and Alley, F.C. 1994. *Air Pollution Control: A Design Approach. 2nd ed*. Prospect Heights, Illinois: Waveland Press, Inc.

Devore, Jay L. 1995. *Probability and Statistics for Engineering and the Sciences. 4th ed*. Pacific Grove, CA: Brooks/Cole.

The Engineering Toolbox. 2003. *Saturated Steam Table in SI Units*. <http://www.engineeringtoolbox.com/28_101.html>. Last accessed December 18, 2003.

Holman, J.P. 2001. *Experimental Methods for Engineers. 7th ed*. Boston, MA: McGraw Hill.

International Standards Organization (ISO). 1995. *Guide to the Expression of Uncertainty in Measurement*. Geneva: ISO.

Kline, S.J., and McClintock, F.A. 1953. Describing Uncertainties in Single-Sample Experiments. *Mechanical Engineering*. 75: 3-8.

Kline, S.J. 1985. The Purposes of Uncertainty Analysis. *Journal of Fluids Engineering*. 107: 153-161.

National Institute of Standards and Technology (NIST). 1994. *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*. NIST Technical Note 1297. United States Department of Commerce. Washington, DC: US GPO.

Sorenson, J.W., and Parnell, Calvin B. 1991. *Agricultural Processing Technology*. College Station, TX: Texas A&M University.

U.S. EPA. 1995. *Compilation of Air Pollutant Emission Factors, AP-42, 5th Edition*, Volume I: Stationary Point and Area Sources. Research Triangle Park, NC: U.S. GPO.

USEPA. 1999. *40 CFR Part 50. National Primary and Secondary Ambient Air Quality Standards*. Research Triangle Park, NC: U.S. GPO.

USEPA. 1999. *40 CFR Part 70. National State Operating Permit Programs*. Research Triangle Park, NC: U.S. GPO.

Wang, L. 2000. *A New Engineering Approach to Cyclone Design for Cotton Gins*. M. S. thesis, Agricultural Engineering Dept., Texas A&M University, College Station.

Yegnan, A., D.G. Williamson, A.J. Graettinger. 2002. *Uncertainty Analysis in Air Dispersion Modeling*. Environmental Modeling & Software. 17: 639-649.

# Tables

Table 1. Instrument Specification

| Parameter | Instrument | Systematic Uncertainty |
|---|---|---|
| $W_i$, $W_f$ | Sartortius SC2 (low volume)<br>Mettler Toledo AG balance (high volume) | $1 * 10^{-7}$ g<br>$2 * 10^{-4}$ g |
| Θ (Time) | HOBO data logger | 0.20 min |
| $\Delta P_a$ | Omega PX274 Pressure Transducer<br>+ HOBO cord | 0.075<br>0.1 mA + 3 % |
| $D_o$ | End Mill Specs | 0.025 in |
| $T_a$ | HOBO Weather Station Temperature/RH Smart Sensor | 0.8 °F |
| $P_a$ | HOBO Weather Station Barometric Pressure Smart Sensor | 1 % |
| $RH_a$ | HOBO Weather Station Temperature/RH Smart Sensor | 3 % |
| $P_{sata}$ | Steam Tables | 0.0001 psia |
| $Q_{massflowmeter}$ | Aalborg GFC17 Mass Flowmeter | 1.5 % FS |
| $Q_{LFE}$ | Meriam Instruments Model 50MC2-2 | 0.344 cfm |
| $\Delta P_c$ | Digital Manometer – Dwyer Series 475 Mark III | 0.5 % FS |
| $T_c$ | Davis Perception II | 1 °F |
| $P_c$ | Davis Perception II | 1 % |
| $RH_c$ | Davis Perception II | 5% |
| $P_{satc}$ | Steam Tables | 0.0001 psia |

Table 2. Gravimetric Sampler Sensitivity Analysis for Uncertainty Propagation

| | **Parameter** | **Units** | **TAMU High Volume** | | | **TAMU Low Volume** | | | **EPA Lower Limit High Volume** | | | **EPA Upper Limit High Volume** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | **Nominal Value** | **Uncertainty** | **% of Total Uncertainty** | **Nominal Value** | **Uncertainty** | **% of Total Uncertainty** | **Nominal Value** | **Uncertainty** | **% of Total Uncertainty** | **Nominal Value** | **Uncertainty** | **% of Total Uncertainty** |
| **Mass** | $W_f$ | G | 9.1 | 2.00E-04 | **1.663%** | 10.3013 | 1.00E-07 | **0.0016%** | 9.786 | 2.00E-04 | **1.431%** | 9.832 | 2.00E-04 | **1.655%** |
| | $W_i$ | G | 9.7 | 2.00E-04 | **1.663%** | 10.3 | 1.00E-07 | **0.0016%** | 9.7 | 2.00E-04 | **1.431%** | 9.7 | 2.00E-04 | **1.655%** |
| **Volume** | $\theta$(Time) | Min | 180 | 0.20000 | **0.016%** | 180 | 0.20000 | **0.0088%** | 180 | 0.20000 | **0.0084%** | 180 | 0.20000 | **0.023%** |
| | Q | Cfm | **50.00** | **4.33220** | **96.66%** | **0.589** | **0.06977** | **99.99%** | **39.00** | **4.66991** | **97.13%** | **60.00** | **4.34247** | **96.67%** |
| **Q** | $\Delta P_a$ | in of $H_2O$ | 1.5493 | **0.2260** | **68.50%** | 1.074 | **0.2118** | **69.2%** | 0.9426 | **0.2078** | **82.31%** | 2.2310 | **0.2465** | **56.30%** |
| | $\rho_a$ | Lbs/ft$^3$ | **0.07213** | **0.000736** | **0.335%** | **0.07213** | **0.000736** | **0.185%** | **0.07213** | **0.000736** | **0.176%** | **0.07213** | **0.000736** | **0.480%** |
| | k | | **0.80235** | **0.037300** | **27.83%** | **0.72620** | **0.04761** | **30.62%** | **0.80235** | **0.03730** | **14.64%** | **0.80235** | **0.037300** | **39.88%** |
| **$\rho_a$** | $T_a$ | ° F | 85 | 0.8 | **0.0069%** | 85 | 0.8 | **0.004%** | 85 | 0.8 | **0.0036%** | 85 | 0.8 | **0.0099%** |
| | $P_a$ | Psia | 14.676 | **0.14676** | **0.3277%** | 14.676 | **0.14676** | **0.181%** | 14.676 | **0.14676** | **0.172%** | 14.676 | **0.14676** | **0.4697%** |
| | $RH_a$ | | 0.58 | **0.0174** | **0.0002%** | 0.58 | **0.0174** | **0.0001%** | 0.58 | **0.0174** | **0.0001%** | 0.58 | **0.0174** | **0.0003%** |
| | $P_{sata}$ | Psia | 0.5961 | 0.0001 | **0.000%** | 0.5961 | 0.0001 | **0.000%** | 0.5961 | 0.0001 | **0.000%** | 0.5961 | 0.0001 | **0.000%** |
| **K** | $Q_{LFE}$/ $Q_{massflow}$ | Cfm | 50 | 0.344 | **0.6095%** | 0.5 | 0.00795 | **1.801%** | 50 | 0.344 | **0.321%** | 50 | 0.344 | **0.8735%** |
| | $\Delta P_c$ | in of $H_2O$ | 1.6 | **0.1** | **12.57%** | 0.8 | **0.1** | **27.82%** | 1.6 | **0.1** | **6.616%** | 1.6 | **0.1** | **18.022%** |
| | $D_o$ | inches | 1.5 | 0.025 | **14.31%** | 0.1875 | 0.001 | **0.810%** | 1.5 | 0.025 | **7.527%** | 1.5 | 0.025 | **20.505%** |
| | $\rho_c$ | Lbs/ft$^3$ | **0.07449** | **0.000762** | **0.337%** | **0.07449** | **0.000762** | **0.186%** | **0.07449** | **0.000762** | **0.177%** | **0.07449** | **0.000762** | **0.4824%** |
| **$\rho_c$** | $T_c$ | ° F | 70 | 1 | **0.0115%** | 70 | 1 | **0.0063%** | 70 | 1 | **0.006%** | 70 | 1 | **0.0164%** |
| | $P_c$ | Psia | 14.676 | **0.14676** | **0.325%** | 14.676 | **0.14676** | **0.1797%** | 14.676 | **0.14676** | **0.171%** | 14.676 | **0.14676** | **0.4657%** |
| | $RH_c$ | | 0.5 | **0.025** | **0.0002%** | 0.5 | **0.025** | **0.0001%** | 0.5 | **0.025** | **0.0001%** | 0.5 | **0.025** | **0.0003%** |
| | $P_{satc}$ | psia | 0.36292 | 0.0001 | **0.000%** | 0.36292 | 0.0001 | **0.000%** | 0.36292 | 0.0001 | **0.000%** | 0.36292 | 0.0001 | **0.000%** |

Table 3. Total Uncertainty for Gravimetric Sampling Under Normal Conditions

| | **Concentration (μg/m$^3$)** | **Uncertainty (μg/m$^3$)** | **Uncertainty (%)** |
|---|---|---|---|
| TAMU – 1 m$^3$/hr | 69.31 | 8.21 | 11.85 |
| TAMU – 39 cfm | 69.22 | 8.41 | 12.15 |
| TAMU – 50 cfm | 69.06 | 6.09 | 8.81 |
| TAMU – 60 cfm | 69.06 | 5.08 | 7.36 |

## Figures

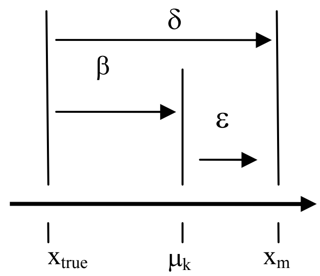


Figure 1. Illustration of Total Error, δ

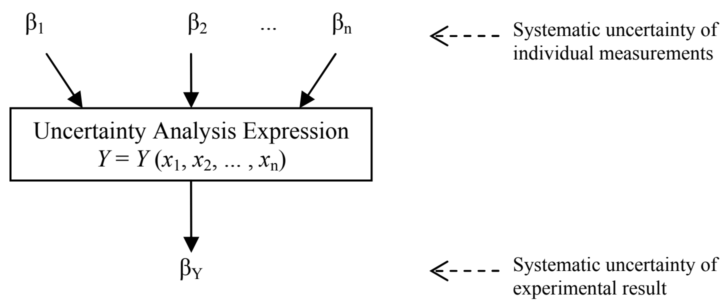


Figure 2. Determining the systematic uncertainty for an experiment (from Coleman & Steele, 1999)

$$C = \frac{W}{V} \qquad (6)$$

$$W = 0.16 * \left(W_f - W_i\right) \qquad (7)$$

$$V = Q * \Theta \qquad (8)$$

$$Q = 5.976 * k * \left(D_0\right)^2 * \sqrt{\frac{\Delta P_a}{\rho_a}} \qquad (9)$$

$$\rho_a = \left[\frac{P_a - RH * P_s}{0.37 * \left(460 + T\right)}\right] + \left[\frac{RH * P_s}{0.596 * \left(460 + T\right)}\right] \qquad (10)$$

$$k = \frac{Q_{IFB}}{5.976 * \left(D_0\right)^2 * \sqrt{\frac{\Delta P_c}{\rho_c}}} \qquad (11)$$

$$\rho_c = \left[\frac{P_c - RH * P_s}{0.37 * \left(460 + T\right)}\right] + \left[\frac{RH * P_s}{0.596 * \left(460 + T\right)}\right] \qquad (10)$$

Figure 3. Breakdown of Equations

$C = W/V$ 69.06 µg/m$^3$

$\omega_c$ 6.0862E-06 g/m$^3$ **6.09 µg/m$^3$** OR **8.81%**

| | | | | % of TOTAL Uncertainty |
|---|---|---|---|---|
| $\omega_W$ | 0.00028284 g | δC/δW | 0.00392385 | 3.3253% |
| $\omega_V$ | 22.0831655 m$^3$ | δC/δV | -2.7098E-07 | 96.6747% |

$W = W_f - W_i$ 0.0176 g

$\omega_W$ 0.000282843 g

| | | | | % of Partial Uncertainty | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{Wf}$ | 2.00E-04 g | δW/δW$_f$ | 1 | 50.0000% | 1.6626% |
| $\omega_{Wi}$ | 2.00E-04 g | δW/δW$_i$ | -1 | 50.0000% | 1.6626% |

$V = Q\Theta$ 8999.996854 ft$^3$ 254.8515 m$^3$

$\omega_V$ 779.8596285 ft$^3$ 22.08317 m$^3$

| | | | | % of Partial Uncertainty | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_\Theta$ | 0.2 min | δV/δΘ | 49.9999825 | 0.0164% | 0.0159% |
| $\omega_Q$ | 4.332197288 cfm | δV/δQ | 180 | 99.9836% | 96.6588% |

$Q = 5.976 * k * D_o^2 * \text{sqrt}(\Delta Pa/\rho a)$ (Uncertainty in Do is already accounted for in the calibration of k)

49.99998 cfm (assume $F_A = 1$)

$\omega_Q$ 4.332197 cfm

| | | | | % of Partial Uncertainty | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{Do}$ | 0 (accounted for in k) | δQ/δDo | 66.6666 | 0.0000% | 0.0000% |
| $\omega_{\Delta Pa}$ | 0.22601105 in of $H_2O$ | δQ/δΔPa | 16.136 | 70.8658% | 68.4980% |
| $\omega_k$ | 0.03729956 | δQ/δk | 62.3173 | 28.7878% | 27.8259% |
| $\omega_{\rho a}$ | 0.00073572 lbs/ft$^3$ | δQ/δρa | -346.599 | 0.3465% | 0.3349% |

$k = Qlfe / (5.976 * D_o^2 * \text{sqrt}(\Delta P_c/\rho_c))$

0.80234559

$\omega_k$ 0.03729956

| | | | | % of Partial Uncertainty | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{Qlfe}$ | 0.344 cfm | δk/δQlfe | 0.01605 | 2.1902% | 0.6095% |
| $\omega_{Do}$ | 0.025 in | δk/δDo | -1.06979 | 51.4130% | 14.3061% |
| $\omega_{\Delta Pc}$ | 0.1 in of $H_2O$ | δk/δΔPc | -0.25073 | 45.1872% | 12.5737% |
| $\omega_{\rho c}$ | 0.00076168 lbs/ft$^3$ | δk/δρc | 5.3857 | 1.2095% | 0.3366% |

$\rho_c = ((P_c - (RH_c * P_{satc})) / (0.37 (460 + T_c))) + ((RH_c * P_{satc}) / (0.596 (460 + T_c)))$

0.07448848 lbs/ft$^3$

$\omega_{\rho c}$ 0.00076168 lbs/ft$^3$

| | | | | % of Partial Uncertainty | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{RHc}$ | 0.025 | δρc/δRHc | -0.0007 | 0.0531% | 0.0002% |
| $\omega_{Psatc}$ | 0.0001 psia | δρc/δPsatc | -0.00097 | 0.0000% | 0.0000% |
| $\omega_{Pc}$ | 0.14676 psia | δρc/δPc | 0.0051 | 96.5422% | 0.3249% |
| $\omega_{Tc}$ | 1 ° F | δρc/δTc | -0.00014 | 3.4047% | 0.0115% |

$\rho_a = ((P_a - (RH_a * P_{sata}))) / (0.37 (460 + T_a)) + ((RH_a * P_{sata}) / (0.596 (460 + T_a)))$

0.07212942 lbs/ft$^3$

$\omega_{\rho a}$ 0.00073572 lbs/ft$^3$ % of Total Uncertainty

| | | | | % of Partial Uncertainty | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{RHa}$ | 0.0174 | δρa/δRHa | -0.00112 | 0.0703% | 0.0002% |
| $\omega_{Psata}$ | 0.0001 psia | δρa/δPsata | -0.00109 | 0.0000% | 0.0000% |
| $\omega_{Pa}$ | 0.14676 psia | δρa/δPa | 0.00496 | 97.8587% | 0.3277% |
| $\omega_{Ta}$ | 0.8 ° F | δρa/δTa | -0.00013 | 2.0711% | 0.0069% |

Figure 4. TAMU – 50 cfm – Uncertainty Analysis

$C = W/V$ 69.31 $\mu g/m^3$

$\omega_c$ 8.2124E-06 $g/m^3$ **8.21 $\mu g/m^3$** OR **11.85%**

| | | | | % of TOTAL Uncertainty |
|---|---|---|---|---|
| $\omega_W$ | 1.41421E-07 g | $\delta C/\delta W$ | 0.333205773 | 0.0033% |
| $\omega_V$ | 0.355610452 $m^3$ | $\delta C/\delta V$ | -2.3093E-05 | 99.9967% |

$W = W_f - W_i$ 0.000208 g

$\omega_W$ 1.41421E-07 g

| | | | | % of Partial Uncertaint | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{wf}$ | 1.00E-07 g | $\delta W/\delta W_f$ | 1 | 50.0000% | 0.0016% |
| $\omega_{wi}$ | 1.00E-07 g | $\delta W/\delta W_i$ | -1 | 50.0000% | 0.0016% |

$V = Q\Theta$ 105.9845585 $ft^3$ 3.001148 $m^3$

$\omega_V$ 12.5582646 $ft^3$ 0.35561 $m^3$

| | | | | % of Partial Uncertaint | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_\Theta$ | 0.2 min | $\delta V/\delta\Theta$ | 0.588803103 | 0.0088% | 0.0088% |
| $\omega_Q$ | 0.069765 cfm | $\delta V/\delta Q$ | 180 | 99.9912% | 99.9879% |

$Q = 5.976 * k * D_o^2 * sqrt(\Delta Pa/\rho a)$ (Uncertainty in Do is already accounted for in the calibration of k)

49.99998 cfm (assume $F_A = 1$)

$\omega_Q$ 4.332197 cfm

| | | | | % of Partial Uncertaint | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{Do}$ | 0 (accounted for in k) | $\delta Q/\delta Do$ | 66.6666 | 0.0000% | 0.0000% |
| $\omega_{\Delta Pa}$ | 0.22601105 in of $H_2O$ | $\delta Q/\delta\Delta Pa$ | 16.136 | 70.8658% | 68.4980% |
| $\omega_k$ | 0.03729956 | $\delta Q/\delta k$ | 62.3173 | 28.7878% | 27.8259% |
| $\omega_{\rho a}$ | 0.00073572 $lbs/ft^3$ | $\delta Q/\delta\rho a$ | -346.599 | 0.3465% | 0.3349% |

$k = Qlfe / (5.976 * D_o^2 * sqrt(\Delta P_c/\rho_c))$

0.80234559

$\omega_k$ 0.03729956

| | | | | % of Partial Uncertaint | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{Qlfe}$ | 0.344 cfm | $\delta k/\delta Qlfe$ | 0.01605 | 2.1902% | 0.6095% |
| $\omega_{Do}$ | 0.025 in | $\delta k/\delta Do$ | -1.06979 | 51.4130% | 14.3061% |
| $\omega_{\Delta Pc}$ | 0.1 in of $H_2O$ | $\delta k/\delta\Delta Pc$ | -0.25073 | 45.1872% | 12.5737% |
| $\omega_{\rho c}$ | 0.00076168 $lbs/ft^3$ | $\delta k/\delta\rho c$ | 5.3857 | 1.2095% | 0.3366% |

$\rho_c = ((P_c - (RH_c * P_{satc})) / (0.37 (460 + T_c))) + ((RH_c * P_{satc}) / (0.596 (460 + T_c)))$

0.07448848 $lbs/ft^3$

$\omega_{\rho c}$ 0.00076168 $lbs/ft^3$

| | | | | % of Partial Uncertainty | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{RHc}$ | 0.025 | $\delta\rho c/\delta RHc$ | -0.0007 | 0.0531% | 0.0002% |
| $\omega_{Psatc}$ | 0.0001 psia | $\delta\rho c/\delta Psatc$ | -0.00097 | 0.0000% | 0.0000% |
| $\omega_{Pc}$ | 0.14676 psia | $\delta\rho c/\delta Pc$ | 0.0051 | 96.5422% | 0.3249% |
| $\omega_{Tc}$ | 1 ° F | $\delta\rho c/\delta Tc$ | -0.00014 | 3.4047% | 0.0115% |

$\rho_a = ((P_a - (RH_a * P_{sata}))) / (0.37 (460 + T_a)) + ((RH_a * P_{sata}) / (0.596 (460 + T_a)))$

0.07212942 $lbs/ft^3$

$\omega_{\rho a}$ 0.00073572 $lbs/ft^3$ % of Total Uncertainty

| | | | | % of Partial Uncertaint | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{RHa}$ | 0.0174 | $\delta\rho a/\delta RHa$ | -0.00112 | 0.0703% | 0.0002% |
| $\omega_{Psata}$ | 0.0001 psia | $\delta\rho a/\delta Psata$ | -0.00109 | 0.0000% | 0.0000% |
| $\omega_{Pa}$ | 0.14676 psia | $\delta\rho a/\delta Pa$ | 0.00496 | 97.8587% | 0.3277% |
| $\omega_{Ta}$ | 0.8 ° F | $\delta\rho a/\delta Ta$ | -0.00013 | 2.0711% | 0.0069% |

Figure 5. TAMU – 1 $m^3$/hr – Uncertainty Analysis

C = W/V 69.23 μg/m$^3$

$\omega_c$ 8.41E-06 g/m$^3$ **8.41 μg/m$^3$** OR **12.15%**

| | | | | % of TOTAL Uncertainty |
|---|---|---|---|---|
| $\omega_W$ | 0.00028284 g | δC/δW | 0.00503058 | 2.8623% |
| $\omega_V$ | 23.8037135 m$^3$ | δC/δV | -3.482E-07 | 97.1377% |

W = $W_f$ - $W_i$ 0.01376 g

$\omega_W$ 0.000282843 g

| | | | | % of Partial Uncertaint | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{wf}$ | 2.00E-04 g | δW/δ$W_f$ | 1 | 50.0000% | 1.4312% |
| $\omega_{wi}$ | 2.00E-04 g | δW/δ$W_i$ | -1 | 50.0000% | 1.4312% |

V = QΘ 7020.005311 ft$^3$ 198.784 m$^3$

$\omega_V$ 840.6202073 ft$^3$ 23.8037 m$^3$

| | | | | % of Partial Uncertaint | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_\Theta$ | 0.2 min | δV/δΘ | 39.0000295 | 0.0086% | 0.0084% |
| $\omega_Q$ | 4.669911 cfm | δV/δQ | 180 | 99.9914% | 97.1293% |

Q = 5.976 * k * $D_o^2$ * sqrt(ΔPa/ρa) (uncertainty in $D_o$ is already accounted for in the calibration of k)

39 cfm (assume $F_A$ = 1)

$\omega_Q$ 4.66991 cfm

| | | | | % of Partial Uncertain | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{Do}$ | 0 (accounted for in k) | δQ/δDo | 52 | 0.0000% | 0.0000% |
| $\omega_{\Delta Pa}$ | 0.20780962 in of $H_2O$ | δQ/δΔPa | 20.6872 | 84.7457% | 82.3129% |
| $\omega_k$ | 0.03729956 | δQ/δk | 48.6075 | 15.0729% | 14.6402% |
| $\omega_{\rho a}$ | 0.00073572 lbs/ft$^3$ | δQ/δρa | -270.35 | 0.1814% | 0.1762% |

k = Qlfe / (5.976 * $D_o^2$ * sqrt($\Delta P_c/\rho_c$))

0.80234559

$\omega_k$ 0.03729956

| | | | | % of Partial Uncertain | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{Qlfe}$ | 0.344 cfm | δk/δQlfe | 0.01605 | 2.1902% | 0.3207% |
| $\omega_{Do}$ | 0.025 in | δk/δDo | -1.0698 | 51.4130% | 7.5270% |
| $\omega_{\Delta Pc}$ | 0.1 in of $H_2O$ | δk/δΔPc | -0.2507 | 45.1872% | 6.6155% |
| $\omega_{\rho c}$ | 0.00076168 lbs/ft$^3$ | δk/δρc | 5.3857 | 1.2095% | 0.1771% |

$\rho_c$ = (($P_c$ - ($RH_c$ * $P_{satc}$)) / (0.37 (460 + $T_c$))) + (($RH_c$ * $P_{satc}$) / (0.596 (460 + $T_c$)))

0.07448848 lbs/ft$^3$

$\omega_{\rho c}$ 0.00076168 lbs/ft$^3$

| | | | | % of Partial Uncertainty | % of Total Uncertaint |
|---|---|---|---|---|---|
| $\omega_{RHc}$ | 0.025 | δρc/δRHc | -0.0007 | 0.0531% | 0.0001% |
| $\omega_{Psatc}$ | 0.0001 psia | δρc/δPsatc | -0.001 | 0.0000% | 0.0000% |
| $\omega_{Pc}$ | 0.14676 psia | δρc/δPc | 0.0051 | 96.5422% | 0.1710% |
| $\omega_{Tc}$ | 1 ° F | δρc/δTc | -0.0001 | 3.4047% | 0.0060% |

$\rho_a$ = (($P_a$ - ($RH_a$ * $P_{sata}$))) / (0.37 (460 + $T_a$)) + (($RH_a$ * $P_{sata}$) / (0.596 (460 + $T_a$)))

0.07212942 lbs/ft$^3$

$\omega_{\rho a}$ 0.00073572 lbs/ft$^3$ % of Total Uncertainty

| | | | | % of Partial Uncertain | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{RHa}$ | 0.0174 | δρa/δRHa | -0.0011 | 0.0703% | 0.0001% |
| $\omega_{Psata}$ | 0.0001 psia | δρa/δPsata | -0.0011 | 0.0000% | 0.0000% |
| $\omega_{Pa}$ | 0.14676 psia | δρa/δPa | 0.00496 | 97.8587% | 0.1724% |
| $\omega_{Ta}$ | 0.8 ° F | δρa/δTa | -0.0001 | 2.0711% | 0.0036% |

Figure 6. TAMU – 39 cfm – Uncertainty Analysis

$C = W/V$ 69.06 μg/m³

$\omega_c$ 5.0836E-06 g/m³ **6.08 μg/m³** OR **7.36%**

| | | | | % of TOTAL Uncertainty |
|---|---|---|---|---|
| $\omega_W$ | 0.000282843 g | δC/δW | 0.003269877 | 3.3099% |
| $\omega_V$ | 22.13631212 m³ | δC/δV | -2.2582E-07 | 96.6901% |

$W = W_f - W_i$ 0.02112 g

$\omega_W$ 0.000282843 g

| | | | | % of Partial Uncertainty | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{wf}$ | 2.00E-04 g | δW/δW$_f$ | 1 | 50.0000% | 1.6549% |
| $\omega_{wi}$ | 2.00E-04 g | δW/δW$_i$ | -1 | 50.0000% | 1.6549% |

$V = Q\Theta$ 10799.99901 ft³ 305.8219 m³

$\omega_V$ 781.7364849 ft³ 22.13631 m³

| | | | | % of Partial Uncertainty | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_\Theta$ | 0.2 min | δV/δΘ | 59.99999448 | 0.0236% | 0.0228% |
| $\omega_Q$ | 4.3424688 cfm | δV/δQ | 180 | 99.9764% | 96.6673% |

$Q = 5.976 * k * D_o^2 * \mathrm{sqrt}(\Delta Pa/\rho a)$ (uncertainty in $D_o$ is already accounted for in the calibration of k)

59.99999 cfm (assume $F_A$ = 1)

$\omega_Q$ 4.342469 cfm

| | | | | % of Partial Uncertainty | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{Do}$ | 0 (accounted for in k) | δQ/δDo | 79.99999 | 0.0000% | 0.0000% |
| $\omega_{\Delta Pa}$ | 0.246462198 in of $H_2O$ | δQ/δΔPa | 13.44669 | 58.2449% | 56.3038% |
| $\omega_k$ | 0.037299561 | δQ/δk | 74.78074 | 41.2585% | 39.8835% |
| $\omega_{\rho a}$ | 0.000735715 lbs/ft³ | δQ/δρa | -415.919 | 0.4966% | 0.4800% |

$k = Qlfe / (5.976 * D_o^2 * \mathrm{sqrt}(\Delta P_c/\rho_c))$

0.802345587

$\omega_k$ 0.037299561

| | | | | % of Partial Uncertaint | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{Qlfe}$ | 0.344 cfm | δk/δQlfe | 0.016047 | 2.1902% | 0.8735% |
| $\omega_{Do}$ | 0.025 in | δk/δDo | -1.06979 | 51.4130% | 20.5053% |
| $\omega_{\Delta Pc}$ | 0.1 in of $H_2O$ | δk/δΔPc | -0.25073 | 45.1872% | 18.0222% |
| $\omega_{\rho c}$ | 0.000761678 lbs/ft³ | δk/δρc | 5.385702 | 1.2095% | 0.4824% |

$\rho_c = ((P_c - (RH_c * P_{satc})) / (0.37\,(460 + T_c)) + ((RH_c * P_{satc}) / (0.596\,(460 + T_c)))$

0.074488482 lbs/ft³

$\omega_{\rho c}$ 0.000761678 lbs/ft³

| | | | | % of Partial Uncertainty | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{RHc}$ | 0.025 | δρc/δRHc | -0.0007 | 0.0531% | 0.0003% |
| $\omega_{Psatc}$ | 0.0001 psia | δρc/δPsatc | -0.00097 | 0.0000% | 0.0000% |
| $\omega_{Pc}$ | 0.14676 psia | δρc/δPc | 0.005099 | 96.5422% | 0.4657% |
| $\omega_{Tc}$ | 1 ° F | δρc/δTc | -0.00014 | 3.4047% | 0.0164% |

$\rho_a = ((P_a - (RH_a * P_{sata})) / (0.37\,(460 + T_a)) + ((RH_a * P_{sata}) / (0.596\,(460 + T_a)))$

0.072129422 lbs/ft³

$\omega_{\rho a}$ 0.000735715 lbs/ft³ % of Total Uncertainty

| | | | | % of Partial Uncertaint | % of Total Uncertainty |
|---|---|---|---|---|---|
| $\omega_{RHa}$ | 0.0174 | δρa/δRHa | -0.00112 | 0.0703% | 0.0003% |
| $\omega_{Psata}$ | 0.0001 psia | δρa/δPsata | -0.00109 | 0.0000% | 0.0000% |
| $\omega_{Pa}$ | 0.14676 psia | δρa/δPa | 0.004959 | 97.8587% | 0.4697% |
| $\omega_{Ta}$ | 0.8 ° F | δρa/δTa | -0.00013 | 2.0711% | 0.0099% |

Figure 7. TAMU – 60 cfm – Uncertainty Analysis

## Appendix A
## Sensitivity Coefficient Determination

$$C = \frac{W}{V} \text{ (refer to equation 6)}$$

$$\frac{\delta C}{\delta W} = \frac{1}{V}$$

$$\frac{\delta C}{\delta V} = -\frac{W}{V^2}$$

$$W = W_f - W_i \text{ (refer to equation 7)}$$

$$\frac{\delta W}{\delta W_f} = 1$$

$$\frac{\delta W}{\delta W_i} = -1$$

$$V = Q * \Theta \text{ (refer to equation 8)}$$

$$\frac{\delta V}{\delta Q} = \Theta$$

$$\frac{\delta V}{\delta \Theta} = Q$$

$$Q = 5.976 * k * (D_0)^2 * \sqrt{\frac{\Delta P_a}{\rho_a}} \text{ (refer to equation 9)}$$

$$\frac{\delta Q}{\delta k} = 5.976 * (D_0)^2 * \sqrt{\frac{\Delta P_0}{\rho_a}}$$

$$\frac{\delta Q}{\delta D_0} = 11.952 * k * (D_0) * \sqrt{\frac{\Delta P_0}{\rho_a}}$$

$$\frac{\delta Q}{\delta \Delta P_0} = 2.988 * k * (D_0)^2 * \sqrt{\frac{1}{\Delta P_0 * \rho_a}}$$

$$\frac{\delta Q}{\delta \rho_a} = -2.988 * k * (D_0)^2 * \sqrt{\frac{\Delta P_0}{(\rho_a)^3}}$$

$$\rho_a = \left[\frac{P_a - RH * P_s}{0.37 * (460 + T)}\right] + \left[\frac{RH * P_s}{0.596 * (460 + T)}\right] \text{ (refer to equation 10)}$$

$$\frac{\delta \rho_a}{\delta RH_a} = \frac{P_{sa}}{460 + T_a} * \left[\frac{1}{0.596} - \frac{1}{0.37}\right]$$

$$\frac{\delta \rho_a}{\delta P_{sa}} = \frac{RH_a}{460 + T_a} * \left[\frac{1}{0.596} - \frac{1}{0.37}\right]$$

$$\frac{\delta \rho_a}{\delta P_a} = \frac{1}{0.37 * (460 + T_a)}$$

$$\frac{\delta\rho_a}{\delta T_a} = \frac{1}{(460+T_a)^2} * \left[\frac{-P_a}{0.37} + RH_a * P_{sa} * \left[\frac{1}{0.37} - \frac{1}{0.596}\right]\right]$$

$$k = \frac{Q_{LFE}}{5.976*(D_0)^2 * \sqrt{\frac{\Delta P_c}{\rho_c}}}$$ (refer to equation 11)

$$\frac{\delta k}{\delta Q_{LFE}} = \frac{1}{5.976*(D_0)^2 * \sqrt{\frac{\Delta P_c}{\rho_c}}}$$

$$\frac{\delta k}{\delta D_0} = \frac{-2*Q_{LFE}}{5.976*(D_0)^3 * \sqrt{\frac{\Delta P_c}{\rho_c}}}$$

$$\frac{\delta k}{\delta \Delta P_c} = \frac{-\frac{1}{2}*Q_{LFE}}{5.976*(D_0)^2 * \sqrt{\frac{\Delta P_c^{\,3}}{\rho_c}}}$$

$$\frac{\delta k}{\delta \Delta P_c} = \frac{\frac{1}{2}*Q_{LFE}}{5.976*(D_0)^2 * \sqrt{\Delta P_c * \rho_c}}$$